The Scientific World of the Crown of Aragon under James I

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This article[1] seeks to provide a general overview of the cultural landscape during the reign of James I, with a particular focus on science. The term "science" will for these purposes be taken to refer to the wide range of disciplines that are studied today in faculties of science, veterinary science, pharmacology and medicine. Science in the 13th century stemmed – at least for the most part – either directly or indirectly from Arab sources. It was shaped in accordance with the particular philosophical and theological ideology of its proponents, who were essentially split into two main groups: the Averroists and the anti-Averroists. Both groups enjoyed support from across the religious spectrum in the Crown of Aragon territories; in other words, from Christians, Jews and Muslims. For the purposes of this article, it will be necessary to grapple with certain philosophical and theological issues that may, at first glance, seem somewhat unrelated to the subject in question.

The sources and reference works available to us as researchers in this field – that of the Christian and Muslim world of the 13th century's Crown of Aragon – are relatively few and far between. Much the same could have been said of Castilian science if it had not been for Alfonso the Wise, who spearheaded the creation of the immense *Libros del saber de astronomía* ("Books on Astronomy Knowledge") compendium.

We can probably accept as a given the fact that 13th-century Christian science was a pale imitation of its Muslim counterpart. In so saying, I am in no way courting controversy. Indeed, this assertion is actually one of the few areas of agreement over the last 100 years of debate about Spanish science – both Menéndez y Pelayo and Echegaray accepted this to be the case, as have all those who have followed. Consequently, 13th-century Christian academics had two cultural languages: Latin for philosophical and theological concerns and Arabic when dealing with scientific issues. Jewish thinkers used Hebrew and Arabic, and Spanish Muslim scholars, simply Arabic – in spite of the fact that interest in Latin writings or those by Christian authors was notable during this period. This final point is evidenced, for example, by Ibn Said al-Maghribi's remarks about *Kitáb al-Anwa'* (*The Calendar of Cordoba*), penned three centuries earlier by Recemundo and Arib ben Said, in the appendix that he wrote to Ibm Hazm's *Risala fi fald al-Andalus* ("Praise Epistle to al-Andalus").

Centuries of Christian, Jewish and Muslim coexistence on the Iberian Peninsula had not always been problem-free. Indeed, over the course of time, there arose sporadic apologetic

1. An earlier version of this article was published under the title "El mundo cultural de la Corona de Aragón con Jaime I" ["The Cultural World of the Crown of Aragon under James I"], in J. Vernet, *Estudios sobre historia de la ciencia medieval,* Barcelona, Universitat Autònoma de Barcelona, 1979, pp. 71-94.

disputations. These were, on the part of Muslims (for example, Ibn Hazm of Cordoba), of an exceptionally high intellectual calibre from the outset, and they became increasingly sophisticated over time for Jews and Christians. If we compare the debates that took place between the "French monk" (most likely the Abbot of either Cluny or Montecassino) and the theologian al-Bayi, who lived in the Saragossa of Muqtadir (1078), with those that took place two centuries later, we bear witness to the journey taken by Christian intellectuals over this time. Indeed, Robert of Chester and Hermann of Dalmatia's translation of the Koran into Latin in 1141, at the behest of Peter the Venerable, provided Christian theologians with a first-rate resource on which they could draw in their increasingly pressing quest to combat Islam through reason – seeing as how the crusaders (both eastern and western) had proved incapable of defeating it through military means.

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The rise of eastern studies in the 13th century led to the evolution of a new education system that ran alongside those devised under Christianity over the course of the previous few centuries. Whilst Ramon Llull learnt Arabic through his own exertions, the Dominican Order understood that there was a need actively to instruct its members in order that they would be able to minister, spiritually, to Mozarabs deported to Morocco and to Christians taken prisoner by Muslims. Whilst it can be presumed that the latter group would have retained their Romance language, the former – after more than five centuries as part of the Muslim world – would have lost it, thus making it vital to communicate with them in the Arabic of their rulers. Moreover, defence of the Christian Creed would necessitate an attack on that of Muslims. Thus, the offensive-defensive debate became more concrete than it had previously been and, with their greater understanding of the subject (there were by now significant numbers of friars conversant and literate in Arabic), the Christians' arguments were falling increasingly into line with the realities of Islam. This is not to say, of course, that there were not occasional flashes of pure intolerance on one side or the other, such as the incident that cost the life of the Valencian Pere Pascual (1227-1300).

There is evidence of this aforementioned deeper understanding of Muslim issues amongst old-school as well as new-school Arabists. The "new school" developed thanks to the efforts of Ramon de Penyafort (1175-1275) who founded two study centres for Hebrew and Arabic in Tunis and Murcia. Both of these schools, which were established with the help of the kings of Aragon and Castile, had apologetic leanings. Although the existence of any links between the latter school and the madrasah run by Muhammad al-Riquti is debatable, preaching priests (according to Ibn Rasiq) were certainly interested in learning about Muslim science – and translating it into their own language – with the aim of being better equipped to criticise it. It seems improbable that Riquti would have allowed himself to collaborate with these priests in such a quest, although it is possible that he may have permitted them to study in his profane science classes. Ramon Martí (a disciple of Penyafort who died c. 1286) was one of the key teachers in these centres of learning. Martí may well have been one and the same person as the "priest of Marrakech", who took part in an intriguing debate with Ibn Rasiq. These types of debates were a standard facet of day-to-day life at that time and they were played out both face to face as well as by letter. I am, incidentally, fortunate enough to have fleetingly seen some letters, unpublished and in Arabic, that were written by a bishop of Tarragona in the first half of the 13th century. It was via these letters, which are now stored in Morocco, that the bishop debated religious issues with a Sufi Muslim.

It is quite clear that, in order to have been capable of debating these types of subjects, the monks' knowledge of the Arabic language could not have been elementary. Nor, indeed, was it. As evidence of this we can take the book *Vocabulista in Arabico,* attributed to Ramon Martí, which cites terms such as *māristān* – terms that would have been quite exotic in the Arab West of his day. The inclusion of such expressions indicates both the depth of the priests' studies, as well as the extent of the influence of Arab culture in Spain.

On setting out on his chosen path, after having finished his studies, Llull would find that this philosophical grounding emanated from his every pore at the expense of exclusively scientific training

On the advice of Ramon de Penyafort, Llull embarked on his own intellectual journey through the medium of eastern studies. Rather than choosing to join a Dominican school, however, he opted to undertake a private course of study – much as had been standard practice in the 12th century. He found himself a Moorish slave, who we can only presume was a learned man as he proceeded to teach his student the basics of Arab philosophy and culture. On setting out on his chosen path, after having finished his studies, Llull would find that this philosophical grounding emanated from his every pore at the expense of exclusively scientific training.

The depth of understanding achieved by these, our earliest, orientalists is demonstrated by the fact that, regardless of their background, they were capable of hitting Islam where it hurt most: around the one and only miracle of Mohammed's life that is acknowledged by Muslim theologians. This miracle came in the form of the Koran's inimitable style and language, which no human being had yet managed to equal and which proved – according to these theologians – that this book truly contained the word of God. This is what Llull tells us in his *Book of the Gentile and the Three Wise Men:* "Said the Saracen to the Gentile: 'Mohammed was a layman, not versed in letters, and the Koran is the most wondrous dictation that has been or ever will be produced'. Hence, had it not been for the will and the work of God, Mohammed could not have dictated such a wondrous text with such wonderfully arranged words as those in the Koran." And he continues in the same vein in his *The 100 Names of God,* in which he puts forward his counterargument: "I, Ramon Llull, unworthy as I am, wish to endeavour, with the help of God, to write this book that covers better subject matter than the Koran, for, if I write a book with better subject matter than the Koran, it means that another man is capable of writing a book as wondrously dictated as the Koran. And I do this so that we can argue, in our discussions with the Saracens, that the Koran, as wondrous a book as it is, was not dictated by God." It seems clear that Llull was attempting to surpass the Koran itself by writing this work in verse – and very regular verse, as opposed to the simpler rhythmic and rhyming prose of the Koran – when one considers the following assertion: "Putting such subtle subject matter as this in verse presents a greater challenge than writing the Koran in the way it was written." I consider it unlikely that Llull's apologetic argument succeeded in winning over any Muslims, who would have found it difficult to judge the aesthetic merits of a Catalan text. Nonetheless, that does not mean that we are able to make any judgments about the rest of the missionary work undertaken by Llull – nor about the doctrinal works, now lost, that he wrote directly in Arabic and that appear to have survived him in North Africa until at least 1394.

The discussions that took place between our priest of Marrakech (Ramon Martí?) and Ibn Rasiq were of a very different flavour. Firstly, the aforementioned priest evidently possessed a deep understanding of one of the most complex and most "baroque" texts of Arabic literature: Hariri's *Maqāmat*. On the basis that Hariri had set a challenge for his emulators, proposing that they scribe a verse obeying all the same rules as one that the priest went on to insert – something that, according to the priest, had not been achieved in all the many years since Hariri's death in 1122 – he intended to show that the *Maqāmat* could not be emulated and must therefore be a text revealed by God, just like the Koran – something that Ibn Rasiq in fact succeeded in refuting by producing, spontaneously, a composition that met all the requisite conditions. At any rate, there is no doubt that the 13th-century orientalists had an excellent command of Arabic as well as direct access to *all* works written in this language.

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Anti-Jewish debate was played out in very different circumstances to those that bore witness to the anti-Muslim disputation, meaning that it took a more striking – and more public – course. Essentially, because the wealthy classes had emigrated to North Africa (to Tunis in particular) to escape the Christian Reconquest, Christians only had access, within their own territories, to large numbers of Mudejars of limited intellectual value. In order to find debaters capable of holding their own in scholastic discourse, it was therefore necessary to look to the Kingdom of Granada or to the other side of the Mediterranean. By contrast, for want of their own homeland to which they could move and which could protect their faith, Jewish intellectuals had stayed on the Peninsula. There had been a changing of the guard over the course of the previous century, and the Jews had remained under Christian leadership – already having a thorough knowledge of the important contributions of Islam to the culture. As a result of this situation, monks could actually engage in the same type of anti-Averroist dispute that they felt obliged to wage against the intellectuals of Muslim states (for want of worthy adversaries amongst the Mudejars) with Jewish subjects living within their own territories. The Jews saw the issue of bringing religion into line with faith either in much the same way as Averroes, Maimonides and Siger of Brabante or in line with the ideas of al-Ghazali, Nahmanides, Ramon Martí and Ramon Llull. In one camp fought Abraham ben Samuel ben Hasday ha-Levi (died 1240) from Barcelona; in the other, the great rabbi Ramban, Moshe ben Nahman (1194-1270) from Girona, better known as Bonastruc de Porta, and Rashba, Solomon ben Abraham ben Adret (1235-1310) from Barcelona. Another, Aaron ha-Levi also from Barcelona, attempted to bring about a rational reconciliation of these two schools, an approach also adopted, by way of esoteric proceedings, by the Kabbalist School (founded in Girona towards the end of the 12th century), in which Azriel ben Menahem (1160-1238) and Abraham ben Samuel Abulafia (1240-1291) from Saragossa were key players. The Kabbalist School was to play a significant role in the evolution of Judaism.

In addition to these internal disputes, Jews within the Crown of Aragon were compelled to defend their faith in the face of Christians mobilised and led by the Dominicans. One of these, Ramon Martí, amassed in his *Pugio fideo* *adversus judaeos* (1278) a large number of arguments intended to demonstrate the authenticity of Christianity on the basis of the very texts of its enemies. In fact, it is only thanks to Martí that we even know about some of these texts, the originals of which have been lost. In the context of these Christian-Jewish clashes, the dispute that Nahmanides waged in Barcelona in 1263 against the convert Pablo Cristiano stands out, as does the enforced censorship of the Talmud, which was perpetrated in 1264 by the Dominicans Ramon de Penyafort, Ramon Martí, Arnau de Segarra and Pere Janer.

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Our 13th-century forefathers commanded a deep and extensive understanding of eastern philosophy and theology, on which they drew in their development of scholasticism. But scholasticism did not only find its roots in the work of Arabic-to-Latin translators; it also incorporated a wealth of elements directly from the patristic, Hebrew and Greco-Latin traditions. The same thing cannot be said, by contrast, for the field of strict science, in which the West was almost entirely dependent on eastern sources. Aware of this fact, Joaquim Carreras, when studying Arnau de Vilanova's vast collection of works, went to great pains to emphasise that Vilanova's collection of Latin books "was divided into two main bodies of work stemming, respectively, from European-ecclesiastical culture and from eastern-scientific culture." We will focus, now, on this latter tradition, despite the paucity of resources and information available that refer to the Crown of Aragon in the 13th century. In reality, only two illustrious figures – multi-faceted writer Ramon Llull and doctor and naturalist Arnau de Vilanova – give us any true sense, in their writings, of the level of scientific understanding during this period.

In the field of arithmetic, the positional numeration system must already have been very widespread, since otherwise there would be no specific reference in Llull's *Tree of Science* to the art of algorism, in other words to al-Khwarizmi's *Arithmetic* translated from Arabic into Latin in the mid-12th century. The reworked version that has reached modern-day scholars, entitled *Liber alghoarismi de practica arismetrice*, was probably the work of John of Seville. This book uses decimal fractions – though not always the decimal system – and makes no reference to the abacus. It appears that this very work was translated by Gerard of Cremona. Here, Egyptian fractions are used. These are fractions with the numerator 1, as well as the fractions 2/3 and 3/4, the remainder being calculated by adding these fractions. Fractions of this type had already featured in a table in the Rhind papyrus and made their way into the Middle Ages via two routes, both of which seem to point to John of Seville. According to the Byzantine thinker and historian Psellus (1018-1078), the academic route can be traced to Anatoly of Alexandria (died 269) and Diophantus, who wrote treatises on the Egyptian methods of calculation. The popular route was via the papyri of Michigan and Akhmin, the Coptic ostraca of Wadi Sarga and from the Koran itself. It was thanks to the aforementioned Hispanic versions and the works of Fibonacci that these fractions spread across the rest of Europe.

Calculations using sexagesimal fractions are of equal interest. These fractions remain essential today in the field of astronomy. Al-Khwarizmi laid down some rules (*Algorismus de minutiis*), which were quickly introduced into European university teaching thanks to *Algoritmi de numero Indorum* (which also stems from his work *Arithmetic*) and, above all, to John of Seville.

There is greater interest in Llull's endeavours in his *Ars* from 1271. These essentially amount to a series of mechanical processes so closely linked to combinatorics that Leibniz and Sebastián Izquierdo regard Llull as one of the forerunners of this particular branch of mathematics. Strictly speaking, this is not accurate. The Majorcan's ideas on this issue may have been clear, but they were rudimentary. The origin of the geometric figures he used has been traced back – by Ribera and Asín – to the work of the Murcian mystic Ibn Arabi. There are other names that figure on this popular Muslim route, such as the occultist al-Buni. Millàs, for his part, believed that it could be related to the world of the Kabbalah, which was beginning to take root in the lands of Catalonia during the 13th century. However, it is worth bearing in mind that the Kabbalah – as well as the gemiatric and mantic systems of divination – were rooted in the great culture of Islam. Ibn Khaldun's work *Muqaddimah* contains descriptions of mechanical processes capable of giving a response – even in verse – to any question that is put. Two such procedures are the mathematical-geometric process by the name of *zairja*, developed in North Africa under the aegis of the Tunisian mystic al-Sadali, and the numerical process developed by al-Sabti. We have been able to place the emergence of the latter somewhere between 1253 and 1269 on the basis of references to ruling sovereigns at the time it was written down. The chronological links between the advent of one system and the other are surprising.

Llull could be related to the world of the Kabbalah, which was beginning to take root in the lands of Catalonia during the 13th century

With these popular processes in mind or as a result of his own mathematical study of similar processes, Abraham ibn Ezra (died c. 1167) already knew that the number of combinations of *n* outcomes from *m* objects equals the number of combinations of *m-n* outcomes from *m* objects. In other words, the following complement formula applies:

$$\binom{m}{n} = \binom{m-n}{n}$$

These formulae – which imply that the coefficients are symmetrical with regard to a vertical axis – hint that mathematicians in the 12th century were already familiar with what we inaccurately term Pascal's triangle or Tartaglia's triangle. Indeed, this triangle is already correctly set out in the works of al-Kasi (died 1429), Omar Khayyam (died 1123) and al-Samaw'al (died 1175), the latter admitting that he took the triangle from a now lost work by al-Karayi (died c. 1019). The fact that the work in which the author sets out Tartaglia's triangle was known to Abraham ibn Ezra is not of particular note when considering that al-Samaw'al – strictly contemporary with the Toledan Ezra – was a Jew who converted to Islam when an old man; that his father, Rabbi Yehuda ben Abun (Yahyá ben ᶜAbbās al-Magribi in Arabic) hailed from Fez, later emigrating to Baghdad; and that Abraham ibn Ezra's son, Isaac, spent some time in his younger days in the capital of the Caliphate (c. 1143) where he studied with some of the leading scholars of his time. His system for calculating coefficients was based on the following addition formula:

$$\binom{m}{n} + \binom{m}{n-1} = \binom{m+1}{n}$$

According to al-Karayi, this can be used consecutively without any limit and the triangle he presents in his work is cut in half because of the symmetry of the coefficients.

Llull gives solutions for the following cases (among others):

$$\binom{16}{2} = 120; \binom{28}{2} = 378; \binom{9}{3} = 84, \text{etc.}$$

He does not explain at all the approach he took – which may be regarded as purely empirical – though one cannot discount the possibility that he had had access to the works of Arab thinkers that had gone before him in this field. However, rather than adopt the approach that these thinkers took – in other words, developing simple permutations and combinations so that they might deduce the essential truths of their art (what we know today as the general formulae) – Llull used a more descriptive, rudimentary and visible system of circles and tables, simplifying and Christianising the *zairja* processes, which were ultimately to filter through into European thought.

Another area of mathematics that must have been given consideration was the question of arithmetic and geometric progressions. In his *Tree*, Llull sets out geometric progressions with ratio 2 (as below):

1	2	4	8
1	3	6	12

These clearly contrast with the arithmetic progressions.

He probably came upon this idea as a result of the posology problems that were particularly in vogue at that time, essentially from the moment that Gerard of Cremona had translated al-Kindi's work *De medicinarum compositarum gradibus investigandi libellus.* This work begins to introduce the notion of psychophysics into medicine since it endeavours to ascertain the efficacy of drugs at different stages of diseases. The theory advanced is that, if the stimulus (drug) dosage increases in accordance with the succession of natural numbers, the ratio between two successive numbers tends towards 1, since the ratio between 1 and 2 is 1/2; between 2 and 3, it is 2/3; between 3 and 4, it is 3/4… and the series 0.5, 0.66, 0.75… does not retain constant ratios between the terms and, at a certain point, the differences become imperceptible.

By contrast, the differences 0.25 (=4), 0.5 (=2)… do retain the same internal ratios (between themselves) as the stimulus. As such, according to al-Kindi, we can establish a parallel between the drug and its effect in accordance with the following gradation:

Sensation		1	2	3	4
Drug	1	2	4	8	16

This is essentially tantamount to stating Weber's (1795-1878) Law: "The heightening of a sensation according to an arithmetic progression is produced by the corresponding increase of the stimulus as a geometric progression." It is also very close to Fechner's (1801-1887) Law, according to which "the sensation is proportional to the logarithm of the stimulus."

Al-Kindi's ideas were compiled and taken on by Arnau de Vilanova, Bernard of Gordon and the Barcelonan Antoni Ricart (died 1422). By contrast, Averroes, and subsequently Pedro Abano, opted instead for an arithmetic progression with ratio 1 on the basis of other mathematical considerations and on the grounds that there was some analogy between these and musical tones.

Nevertheless, the ratio approach that took root among medieval writers was al-Kindi's, which was not only suited to expressing the ratio between the stimulus and the sensation, but also seemed appropriate to represent variable movement where velocity is the fluid element. Bradwardine, in his analysis of a moving object's speed as the ratio between force and resistance, was ultimately to obtain the same series as the posologists.

In the specific discipline of geometry, we immediately come across Llull's (*New Geometry*), the explanations set out therein being partially complemented by those given in *Tree*. When considering this as a whole, one is not left with a terribly favourable impression. Rather, it demonstrates that this is merely an endeavour to popularise scientific concepts with which 13th-century scholastics were already well acquainted. Indeed, the Carreras brothers included the Majorcan thinker's work within popular scholasticism, which, in turn, is often tainted by the *mutakallimun* atomistic doctrine. In this regard, Llull's opposition to the philosophers Avicenna and Averroes leads him, in *Tree*, to accept the existence of indivisible points, space and time. He does not, however, devote any attention to the implications that accepting one theory or the other would have in terms of the casuistry of problems of movement (he must have been aware of these problems as a reader of al-Ghazali's *Maqasid al-falasifa* ("The Intentions of the Philosophers") – which he partially summarised in Catalan verse – as well as of Avicenna and Averroes). Although occasionally appearing to home in on a generic notion of the infinitely big and small, Llull thus passed up the opportunity to play a genuine part in mathematical speculation on this subject, which had been introduced into Christianity by Abraham bar Hiyya of Barcelona and which, with subsequent reformulations and via Campanus of Novara, Saint Thomas, Bradwardine *et al.*, was to enjoy one final coming and major importance in Cavalieri's (1598-1647) indivisibles.

The subsequent definitions he gives for a point and a plane – as well as the variety of shapes given to the points – call to mind the different shapes medieval thinkers attributed to the atoms of elements and of different substances. Moreover, within the wealth of purely geometric knowledge that Llull demonstrates, his efforts to square the circle through *figura magistrales* are particularly noteworthy. His ideas on this subject – set out in his tract entitled *De quadratura et triangulatura circuli* (1299) – went on to influence Nicholas of Cusa and other later thinkers, such as John of Herrera.

The various references by Llull, given the great interest they aroused, led A.E. Nordensköld to suspect that he was in fact the creator of the nautical chart

Llull's *Tractatus novus de astronomia* ("Book of the Seven Planets"), in which there is a consistent blend of astrology and philosophy, has a similarly popular slant. However, both in this work and in the above-mentioned works *New Geometry* and *Tree of Science*, it is possible to glean a series of small nuggets of information that clearly shed light on certain aspects of the history of science. These, in turn, occasionally have parallels in Ramon Llull's Catalan work. Take navigation, for instance. Two texts dating from the second half of the 13th century indicate that initial experiments were underway in the Mediterranean to trial the use of astronomical navigation. The texts are those by Alfonso X the Wise, who tells us that vessels were equipped with a compass for orientation, and the various references by Llull – which, given the great interest they aroused, led A.E. Nordensköld to suspect that he was in fact the creator of the nautical chart (*normal portolan*). This hypothesis was based on the paragraph entitled *De questionibus navigationis* in *Ars Magna Generalis et Ultima*, which has a number of parallels at different points in *Tree* (written in 1295). One such example is where he highlights the fact that sailors had instruments, charts, compasses and the north wind. And, if we take into account the evidence in documentation of that period, this was an accurate representation.

By cross-referencing, we are able to ascertain that Llull is referring to a fuel compass, which was first described by a western Arabian prior to 1229. With regard to the instruments, here he is referring to the astrolabe, quadrant and nocturlabe. All three had been known at least since the 10th century, though we have no direct knowledge of the way in which they were used in navigation. Poulle is credited with being the first to have highlighted the oldest known reference to these instruments to date. This reference is to be found in Raymond of Marseille's essay on the astrolabe (1140), which explains how sailors set their course using a dual observation technique based on the upper and lower altitudes of a circumpolar star, such as Benenas *(Ursa Minor* or Little Bear) or Algedi, which was known as *Maris Stella* (Star of the Sea) because sailors frequently made use of it. Another instrument may well be the nocturlabe, which made it possible to pinpoint the time of night using a method known as the *Pole Star procedure* that involved observing the *Duo Fratres* or Two Guards stars (α and β of *Ursa Minor*). One of the first descriptions – if not the first description of this instrument – was furnished by Llull in *New Geometry*.

There may well already have been two different systems for determining latitude, each used according to the circumstances. One would thus be the Two Guards system, of Indian origin, first – and very belatedly – described by Valentim Fernandes in his *Repertorio dos Tempos* (1518), and the other, a system based on observing the Sun during the daytime. This required not only a table of solar declinations – which had existed since ancient times – but also an almanac charting dates in the year. Such almanacs did in fact exist in the Kingdom of Aragon during the period under examination. Rénaud had already indicated that, in using the word "almanac", the Moroccan astronomer Ibn al-Banna (1256-1221) was in effect alluding to events based on the Sun and Moon, which remain the crucial element in the modern-day nautical almanac. These were presented in precisely that way, for example, in Ammonio's *Unicum*, which was reworked by al-Zarqali in his version prepared for Alfonso and then later reworked in the 1307 *Almanach perpetuum ad inveniendum vera loca planetarum in signis or Almanac de Tortosa*. Thus, the sailors of the period were able to associate the declination of the Sun on a given day with a specific date and, by simply observing the sun's height in the sky, they were able to deduce the latitude of the place they were in based on the well-known formula $\varphi = 90 - h \pm \delta$ when the Sun was at its upper culmination. It also appears certain that they had an almanac since *Vocabulista* gives us the terms *calendarium, manāj, niwarraj, tawrij*. The latter – in its plural form *tawārij* – can be linked to the root *'rj* ("to date") and, as such, yields a definition that characterises the events in an almanac: the fact that they coincide precisely and without exception with a specific date.

Sailors already had a system for estimating distances on a given course, as well as trigonometric tables or a nomogram enabling them to routinely pinpoint their position

The nautical chart had already been known for many years before the period in which Llull wrote. Evidence that this was so comes not only from direct references that he makes to the chart, but also from allusions to a system used to determine distances at sea. Initially, in the first half of the 13th century – when compasses were already in existence – the first local nautical log books were drawn up. These were then incorporated into one general log, the oldest of these being possibly the one published by Motzo in the mid-13th century. By around 1270, sailors were already frequently using logs and nautical charts to help them navigate. Furthermore, in Llull's *Tree*, in the section dealing with the way in which sailors measured nautical miles at sea, we can see that sailors already had a system for estimating distances on a given course, as well as trigonometric tables or a nomogram enabling them to routinely pinpoint their position by dead reckoning if they drifted off course because of unexpected winds or currents. Llull's text sets out the problem with the most straightforward example possible, in other words if the drift, from the point of origin, occurs at an angle of 45°. Despite this, we can clearly see that this constitutes the beginnings of the *toleta de marteloio* system, first committed to paper in the Mediterranean (though it was also known in the Indian Ocean) in a memorandum addressed to the Captain General of Venice in 1428. Here the *toleta de marteloio* appeared in tabular format with quarter points – 11°15', 22°30', 33° 45', 45°, 56°15', 67°30', 78°45' and 90° – corresponding to the first quadrant of the 32 winds of the compass. The system could give answers to the following two types of question: 1) Let's assume I sail eastwards, but winds in the opposite direction force me to sail four quarter points south-eastwards (*eixaloch*). When I have sailed 100 nautical miles with the *eixaloch* wind, how far will I have moved away from the East? What will be the distance to my desired course, the East? 2) Let's assume I have moved 100 nautical miles away from my desired course, the East. How many nautical miles must I sail and how far away to the East will my desired course be if I turn *n* quarter points?

The example used by Llull is the first and most straightforward since he knows the hypotenuse (100 nautical miles) and the angle (45°). However, in this instance, the sine is the same as the cosine, meaning that the distance *actually* travelled eastwards is exactly the same as the distance from his desired course.

In exactly the same way, Jaume Ferrer of Blanes' proposal for establishing the demarcation line between the territories of Spain and Portugal was also based on problems of this nature. In accordance with the Treaty of Tordesillas, he stated that the line should be drawn 370 leagues westwards from a latitude of 15° – in other words, where the Cape Verde Islands lie – and, in an extensive and obscure cosmographical analysis, he went on to calculate the distance in leagues of a longitude of 1° both in the Equator and in the aforementioned latitude. This discussion is rather superfluous in terms of the essence of the problem that the *toleta de marteloio* solves in the first example – that is to say, if a ship veers away from the course it wishes to follow (in this instance, westwards along the 15° line of latitude) because of a given wind (11° 15', 22 °30', etc.), what will the drift be? Since Ferrer knows that the distance to be covered is 370 leagues, he selects a particular wind – in this instance the 11° 15' wind (W 1/4 NW) and then goes on to establish the ship's drift, based on this wind and on the distance of 370 leagues. In other words, the calculation would be D = 370 tan 11° 15' = 73.5974 leagues. Jaume Ferrer rounds this figure up to 74 leagues. However, there is no need to make any great effort in this regard since the *toleta de marteloio* has already solved the problem, as we shall now demonstrate. This table, exactly as it is reproduced in Nordenskjöld's *Periplus*, lists values for the different elements that have to be accounted for when sailing in a low hydrographic basin where the earth is regarded as being flat and where there is only a small margin of error compared to treating the earth as a sphere. It should be borne in mind that, in solving problems involving a plane right-angled triangle, the only pertinent information

Course	c = 100 sin R	b = 100 cos R	$a = \frac{10}{\sin R}$	$b = \frac{10}{\tan R}$
11° 15'	19.509 *20*	98.078 *98*	51.258 *51*	50.273 *50*
22° 30'	38.268 *38*	92.388 *92*	26.131 *26*	24.142 *24*
33° 45'	55.557 *55*	83.147 *83*	17.999 *18*	14.966 *15*
45°	70.710 *71*	70.710 *71*	14.142 *14*	10 *10*
56° 15'	83.147 *83*	55.557 *55*	12.026 *12*	6.681 *6*
67° 30'	92.388 *92*	38.268 *38*	10.824 *11*	4.142 *4*
78° 45'	98.078 *98*	19.509 *20*	10.196 *10*	1.989 *5*
90°	100.000 *100*	0 *00*	10 *8*	0 *00*

to account for is an angle (course) and, potentially, a leg or the hypotenuse, to which the *toleta* ascribes values of 100 or 10 depending on the particular case under examination. If we recalculate the table, we have the following (calculated values in ordinary type, values given in the *toleta* in italics):

Both Ferrer and Llull had the table in front of them. Ferrer tells us that: "[...] the point of the wind that takes the ship along its course, starting from the Cape Verde Islands and after three hundred and seventy leagues, will be seventy-four leagues away from the line of latitude or the Western line by a ratio of twenty per cent [...]" In other words, using the first row and column of the *marteloio*, he establishes the following:

$$\frac{370}{x} = \frac{100}{20} \quad x = 74 \text{ leagues}$$

Here, we are disregarding the expression of these 74 leagues in grades. Given what's outlined above, Llull's text ("and considering the centre of the circle where the winds form angles, and then considering the wind from the east, the ship moves 100 nautical miles from the centre") requires no further comment.

Those who were well-versed in applying Aristotelian natural philosophy to predict the future were always unequivocal in stating that their predictions did not imply any sense of pre-destiny, nor were they an attempt to counter divine omnipotence

In the field of pure astronomy, we ought to highlight Llull's relatively sensible explanation of the apparent increase in the Sun's diameter when near the horizon, as well as his explanation of the stains on the Moon (he views the Moon as essentially a mirror reflecting the Earth's configuration). For the rest, the other theories advanced in this period are a mixture of Aristotelian natural philosophy. Those who were well-versed in applying this philosophy to predict the future – and Llull followed them – were always unequivocal in stating that their predictions did not imply any sense of pre-destiny, nor were they an attempt to counter divine omnipotence.

One of the astronomical phenomena that most readily lent itself to being used for astrological purposes was the eclipse. As such, we see a number of eclipses listed in *The Book of Wisdom*, specifically the following:

21-22.8.1290	Moon in Pisces	Phase 0.66, sea storms
5.9.1290	Sun 20° in Sagittarius	Phase 4 fingers, harm to mankind
24.2.1291	Moon	2 parts, harm
30.07.1292	Moon	0.5
06.07.1293	Sun	
30.05.1295	Moon	0.75 death of a great Christian King

Generally speaking – and assuming of course that the dates of these eclipses were calculated in advance (which might be questionable given the uncanny accuracy of the astrological prediction based on the last eclipse) – we have to acknowledge just how precise the astronomer's forecasts of the eclipses were. Apart from the 1291 eclipse – which actually occurred on the 14th rather than the 24th (the error owing either to a slip by the manuscript's scribe or, more likely, to a printing error, with the inadvertent addition of an X) – the others coincide with what actually happened. Moreover, the predictions of solar eclipses, which are the hardest to forecast, are correct: the 1290 eclipse was annular and never at any point total, while 1291 witnessed a total eclipse ("and the whole Earth will fall into darkness as if it were night"), the central track passing over Spain along the Gulf of Cadiz coast. What we are told about the first eclipse ("there will be a partial eclipse of the sun; in other words, 11 fingers") would certainly have tallied with what we could see at our latitude since the central track ran along the 50° north parallel. Lastly, one can but admire the accuracy of the prediction that is deduced from the 1295 lunar eclipse ("death of a great Christian king"), which is probably a reference to Sancho the Brave (25 April).

In the context of general knowledge, I should perhaps also mention the astronomy texts used by astronomers of the period and indeed by Arnau of Vilanova. These are *De spera* and *Compotus correctorius* written by the English bishop Robert Grosseteste (1168-1253). The first is a compendium of the work John of Holywood – better known as John of Sacrobosco (died c. 1256) – had written on the same subject. It contains a rather accomplished synthesis of the astronomy works written by al-Fargani, al-Battani and Alhacen, all of whom were known thanks to the Arabic-Latin versions produced in Spain. Grosseteste, however, went a step further, adding further information on the trepidation of the equinoxes. He was also responsible, together with Roger Bacon, for developing a new type of astronomy book, the *Theorica planetarum.* This appears to expand on the last section of Sacrobosco's *De spera*, which aims to calculate the size and dimensions of the universe in accordance with the ideas that Ptolemy set out in his *Planetary Hypotheses* and that Campanus of Novara, his greatest advocate during that period, would probably also have been familiar with through the John of Seville version of al-Fargani's work. The method takes as a starting point what is known to be the absolute distance to the nearest heavenly body, the Moon. Thereafter, it is possible to deduce the distances to the other planets provided you assume that the apogee of any one planet's orbit meets the perigee of the planet immediately above it and so on and so forth. In other words, the theory is that space is made up of homocentric spheres and rings that are all intimately connected.

Compotus correctorius aims to bring the solar calendar into line with the lunisolar calendar. In other words, it aims to establish, once and for all, the date of the first day of spring (when the Sun enters Aries) and the rules for determining when Easter falls. Grosseteste is critical of the Metonic cycle (19 Julian years), which was used in both the Christian and Jewish calendars. His rationale is that, if 235 lunar months (6939^{d} 687287) are the same as 19 Julian years (6939^{d} 75), an error occurs. Over 304 years, this error amounts to 1 day, 6 minutes and 40 seconds, with attendant repercussions for calculating the date of Easter. As a result, he advocates reform of the calendar to take into account the exact values for the (tropical) year and the (synodic) month. This observation demonstrates that al-Battani's *Tables* tally closely with the movement of the Sun. In his *Kalendarium,* Grosseteste uses the 76-year Callippus cycle for the lunisolar correlation, while in *Compotus* he advocates using the 30-year Arab cycle with a total of 10,631 days on the grounds because, at the end of this period, the lunations coincide again (360 lunar months being equal to 30 years). In other words, our reading of Arnau de Vilanova's writings prove to what extent there was a tendency to acknowledge the advances of classical Arab culture (the Metonic cycle with its multiples and submultiples). Similarly, other books in his collection dealing with astronomy and astrology probably exhibit the same eastern-leaning bent.

Physics during this period is not of great significance, focusing on questions of movement, which in turn tie in with philosophical considerations, such as the notion of a vacuum, the divisibility of space and time, etc. Nonetheless, from Llull we are able to ascertain that there were already some notions of hydrostatics during this period, with knowledge of the concept of density and, critically, of optics. In this regard, the fact that John Peckham's (died 1292) *Perspectiva Communis* was part of Vilanova's collection indicates that sound information was available since it stems from Ibn al-Haytham's work and, like the work of Bacon and Witelo, provides information about the camera obscura.

There is greater interest and better information surrounding alchemy, although a number of the works on the subject attributed to Llull and some of those attributed to Vilanova are somewhat spurious. At any rate, we know that Llull did not believe in the transmutation of metals. The texts in which he refers to the subject clearly indicate that he was acquainted with – indeed he espoused as his own – the opinion expressed by Avicenna in *Epistola ad Hasen regem* and in *De congelatione et conglutinatione lapidibus,* in which he argues that it is impossible to convert a non-precious metal into gold or silver since all that can be created is a semblance of, or a substitute (*sibga*) for, the precious metal. This substitution or staining was possible thanks to Geber's theory on the sulphur and mercury principles, the idea being that these are not exactly the substances designated with these names, but rather hypothetical substances that call to mind sulphur (because they are hot and cold) or mercury (because they are cold and moist). Hence, "alchemists [...] are not really able to transform the nature of things. Rather, they can effect ostensible changes, such as by dyeing the colour red a silver-like white colour or a gold-like yellow colour [...]," since the characteristics of a given metal are made up not only of the proportions of the sulphur-mercury principles, but rather are also determined by their degree of purity. In this respect, Llull's works generally run counter to Vilanova's and, when the likes of Raymond of Tàrrega (died 1371) and John of Rupescissa (died c. 1356) penned their alchemy treatises in the 14th century, they had to resort to inventing this notion of Llull having been converted to alchemy by Vilanova.

In the inventory detailing Vilanova's possessions after his death, we find, under number 109, a book beginning *Lunam et solem.* We believe this to be one and the same as *Risālat al-sams ilà al-hilāl* by the Egyptian occultist Ibn Umail, known also by the Latin names Senior, Senior Zadith and Zadith ben Hamuel. This book was the subject of a commentary written in Arabic and then translated into Latin with the title *Tabula chimica.* The bibliography that both Llull and Vilanova cited and used in the literature they produced on this subject includes a series of books, such as those written by Morenius and Geber as well as the *Turba Philosophorum.* These books indicate that the material they set out is of eastern origin. Indeed, according to certain authors – though of course not Juan A. Paniagua – Vilanova's inventions in this field are attributable to these books: the preparation of spirits; the production of nitric acid first described in Geber's *Summa perfectionis; aqua regia* (royal water), the origin of which is also to be found in Geber's works; and alcohol. It is also possible that Vilanova is responsible for an adapted translation of the Arabic book written by the Greek alchemist Zosimo.

In this respect, Llull's works generally run counter to Vilanova's and, in the 14th century, they had to resort to inventing this notion of Llull having been converted to alchemy by Vilanova

Evidence of natural sciences during this period is fragmentary and anecdotal. We know, for instance, that Vilanova had a *Lapidary* and that there were a number of treatises on falconry (for example, *Lo libre dell nudriment he de la cura dels ocels* born out of the corresponding section of *Speculum...* by Vincent of Beauvais [died c. 1264]), on hunting hounds and on hippiatrics, such as the Catalan versions of Jordanus Ruffus' *De medicina equorum* and the anonymously written *El libro de los caballos*

("The Book of Horses") produced under the aegis of Alfonso X of Castile.

By contrast, medicine and pharmacology appear far more cohesive. Moreover, we also have much more detailed, sound knowledge about them than we do about other branches of science. This is thanks both to the more general studies produced in the field by the likes of A. Cardoner i Planas and to the more narrowly focused studies, such as those confirmed as the work of Arnau de Vilanova. Indeed, we merely have to recall the Eastern sources underpinning the work of the man responsible for translating Avicenna, Qusta ibn Luqa, al-Kindi, Ibn al-Yazzar, Avenzoar, etc. His library collection also included *Cyrurgia* (*The Surgery of Theodoric*), written by his contemporary Theodoric Borgognoni (1205-1298) and quickly translated into Catalan. At the same time, Esteban of Saragossa was translating Ibn al-Yassar's *The Book of Simples* into Latin (1233) and Abraham of Aragon, a Jew, was acquiring richly deserved fame as an ophthalmologist. We shall not therefore dwell on these matters. We shall, however, mention in passing that *De ornato mulierum* and *De decoratione* – both essays on cosmetics that have been attributed to Vilanova – may have their origins in Ibn Zuhr's *Kitāb al-Zina*. Similarly, *Liber de pronosticationibus sompniorum* by a certain William of Aragon (Arnau de Vilanova perchance?) appears to have been heavily influenced by the Arab oneirologist Ibn Sirin.

Thinking surrounding the word *malastān* (*marastān*, *hospitale*), which appears in *Vocabulista*, perhaps sparks greater interest since it hints that eastern-style hospitals (in other words, those with specialist wards and one devoted to treating the insane) were introduced into the Kingdom of Aragon during this period. As the name suggests, the institution was of Persian origin and the first operational hospital – which brought together Indian and Greek practices – was located in Gundishapur. Caliph Walid I (705-715) introduced the notion of hospitals into Islam and, from the 9th century, the number of hospitals unexpectedly multiplied and they went on to become great institutions. One such example was the Adudi Hospital, opened in 982, which had 80 different physicians from a range of different specialisms in its employ (ophthalmologists, surgeons, traumatologists, etc.), who also performed teaching duties. Nonetheless, even during this period there were independent psychiatric institutions and these were often ultimately called (*bi*)*maristan*. Al-Mubarrad (died 898) is responsible for two anecdotes that demonstrate this. The first relates to a visit to the Dayr Hizqil (or Harqal) asylum and might be interpreted as an adaptation of the Bedouin theme of Maynun (love-sick "madness") for a city-dwelling reader. The second centres on a question of courtesy. Both tales demonstrate how these madmen – sane when they engage in dialogue with the narrator – are chained and shackled. A century on, the great writer al-Hamadani (968-1008) devotes one of his *maqāmas* to an eloquent madman from the asylum in Basra. I have concrete proof that the existence of these institutions was known in Spain at the end of the 10th century, though I have no evidence that they actually existed in Spain before 1367.

The first operational hospital – which brought together Indian and Greek practices – was located in Gundishapur

Initially, the treatment used to control paroxysms of schizophrenia was the same as was practised in the West until Pinel's emergence and essentially amounted to the use of brute force and whips. It subsequently took a more humane turn – we know that Muhaddad al-Din ben Dajwar (1169-1230), an eastern practitioner, treated the insane by adding a suitable dose of opium to their barley water, thus bringing any crisis under control. And, by chance, one of the few drugs cited in *Vocabulista* is *jasjāsa*, which is correctly translated as *papaver* (poppy).

As such, it seems logical that these ideas on how to treat patients suffering from this kind of condition filtered into the Iberian Peninsula during the 13th century, subsequently giving rise to the centres in Granada (1367), Barcelona (1375) and Valencia (1409).

In our view, this brief overview of scientific studies in the 13th century under James I clearly demonstrates that this was no period of decline. It shows that science of eastern origin was alive and well, even if the polarising shift towards the study of the arts over the study of the sciences was already underway. Be that as it may, scientific developments on the other side of the Pyrenees did not differ significantly from those in Spain. Were we to compare that "western" science with this "eastern" science from that period, the former would probably come a poor second. That, however, is not the object of the exercise here, since we have focused our attention entirely on examining the cultural level attained by Christians during this period.

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